A comparison of shaper-based and shaper-free architectures for feedforward compensation of flexible modes

Dan Pilbauer and Wim Michiels and Tomáš Vyhlídal

Delay based reference shaping is a broadly used technique, very easy to implement and with good filtering properties. Guaranteeing the filtering property for input and output disturbances as well, motivated the developments of novel control architectures where delay based shapers appear in feedback loops as inverse shapers. Intuitively, one may ask to what extent the explicit inclusion of shapers is necessary, i.e., whether similar filtering properties could be induced by a suitable parameterization of the controller, which would have as additional advantage that the closed loop system would remain finite-dimensional. The aim of the chapter is to shed a light on this matter and provide answers. For this, a scheme with an inverse shaper is compared with a classical control scheme, where constraints on the location of the controller's zeros are added in the design, in order to induce the filtering property. Both methods are presented with numerical examples, concluding with a discussion of the results. It is show, among others, that filtering is indeed possible by an appropriate controller design, but at the same time, major limitations appear, that further motivate the use of delay based shapers.

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1 Introduction

Techniques modifying a reference input and filtering undesired frequency by a time delay filter (an input shaper) are broadly used. The input shaping idea was firstly proposed in [1] and named ”Posi-cast”’, for a review on input shaping since then see [2]. The architecture using Posi-cast is shown in Fig. 1. The scheme depicts a classical feedback scheme with input shaper S connected on the input reference r and flexible structure F on the output, and P denoting the plant and C the controller. The goal of such a scheme is to compensate oscillatory modes of the flexible structure represented by a couple of oscillatory poles, as a rule, by including the input shaper that compensates the poles by dominant zeros from its infinite spectrum. Since the introduction of the input shaper technique, many modifications of the control scheme have been developed. Modifications with shapers incorporated directly in feedback loop were motivated by filtering not only the reference signal but also external disturbances. A first attempt to develop this scheme was in [1], where a rather complicated scheme with the shaper and compensator is combined. As shown in [3], such a scheme is limited by the controller and the system which both have to be biproper. Later on, the hybrid control approach proposed in [4] combined the Posi-cast principle with a classical feedback system design followed by [5, 6], where the authors anaylse the closedloop stability of systems with shapers within the feedback via root locus plots. The scheme with the shaper within the feedback loop, where the shaper is placed in between controller and the system, is effective only when disturbances appear on the sensor but not on the actuator, see [7]. The novel architecture proposed in [3, 8] suggests to use an inverse shaper in the feedback loop.

Here we show an alternative scheme without shaper and with only one controller. The controller’s parameters are designed with constraints on its zeros which results in similar properties as the scheme with shaper. However, as will be shown, this approach has limited usage.

Firstly in Sec. 2, we introduce feedback architecture for feedforward compensation. This section shows a technique without inverse shaper and describes limitations for this method. A shaper-free method is motivating shaper-based method, described in Sec. 3. Sec. 3 firstly introduces the classical input shaping techniques and continues with inverse shaper based technique. Both, the shaper-free and shaper-based techniques, are compared in numerical simulations in Section 4.

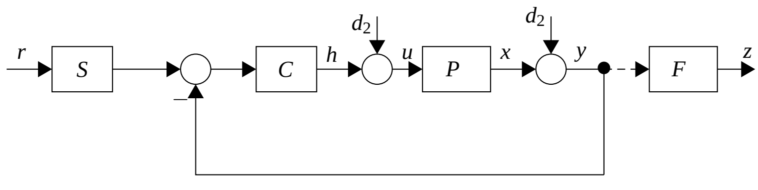

Fig. 1 Classical feedforward application of input shaper

2 Architecture without input shapers

Consider a system with a block scheme depicted in Fig. 2, where system P with strictly proper transfer function $P(s)$, and with input u and output x. System F, the flexible structure, has transfer function $F(s) = \frac{F_N(s)}{F_D(s)}$ with y being the input and z the output. The inputs $d_{1,2}$ are unmeasurable input and output disturbances actuating

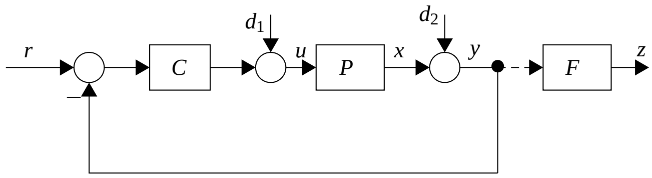

Fig. 2 Block scheme of the classical configuration with controller C, plant P and flexible structure F

on system $P(s) = \frac{P_N(s)}{P_D(s)}$. The controller $C(s) = \frac{C_N(s)}{C_D(s)}$ is assumed to be a fixed-order controller of the form

$$C \begin{cases} \dot{p}(t) = A_c p(t) + B_c(r(t) - y(t)) \\ u(t) = C_c p(t) \end{cases} \tag{1}$$

where capital letters are real-valued matrices of appropriate dimensions. We assume for the moment a single input single output (SISO) system and controller. Therefore (1) is described in the frequency domain by a single transfer function

$$C(s) = C_c(sI - A_c)^{-1} B_c. \tag{2}$$

Closing the loop with the controller and system, the following transfer functions are in the scope of interest. The first transfer function is from reference r to output z

$$T_{zr} = \frac{CP}{1+CP} F = \frac{\frac{C_N P_N}{C_D P_D}}{\frac{C_D P_D + C_N P_N}{C_D P_D}} \frac{F_N}{F_D} = \frac{C_N P_N}{C_D P_D + C_N P_N} \frac{F_N}{F_D} \tag{3}$$

and the transfer functions from disturbances $d_{1,2}$ to output y are

$$T_{zd_1} = \frac{P}{1+CP} F = \frac{\frac{P_N}{P_D}}{\frac{C_D P_D + C_N P_N}{C_D P_D}} \frac{F_N}{F_D} = \frac{C_D P_N}{C_D P_D + C_N P_N} \frac{F_N}{F_D} \tag{4}$$

$$T_{zd_2} = \frac{1}{1+CP} F = \frac{1}{\frac{C_D P_D + C_N P_N}{C_D P_D}} \frac{F_N}{F_D} = \frac{C_D P_D}{C_D P_D + C_N P_N} \frac{F_N}{F_D} \tag{5}$$

in order to compensate the oscillatory pole pole of $F(s)$ by zeros, the transfer function T_{zr} requires the numerator of the controller C_N to have zeros placed on the position of poles of the flexible structure whereas transfer functions T_{zd_1} and T_{zd_2} require denominator C_D to have zeros placed there. These two requirements are contradictory because they cannot be satisfied simultaneously. When the numerator of the controller has required zeros, a change of reference does not excite the oscillatory mode of the flexible structure but the disturbance does. On the other hand, when the denominator of the controller has required zeros, the disturbance does not excite the flexible structure but the change of reference does. As only one one requirement can be satisfied the specific application decides what is the most important. Here, we show how to achieve partial zero placement for controller's numerator.

Consider system P as a linear SISO system described by sets of differential equations

$$P\begin{cases}\dot{x}(t) = A_P x(t) + B_P u(t)\\ y(t) = C_P x(t)\end{cases} \tag{6}$$

where capital letters are real-valued matrices of appropriate dimensions.

The flexible structure F is an oscillatory, low damping system with transfer function $F(s) = \frac{F_N}{F_D}$, where denominator F_D is defined by flexible mode as $s_{1,2} = f(\zeta, \omega)$, where ζ is the damping ratio and ω the natural frequency. The controller C is defined by (1)-(2). To maintain linearity in the design of the parameters the controller is considered in canonical form with matrices

$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, C_c = \begin{bmatrix} c_n \ c_{n-1} \ \cdots \ c_1 \end{bmatrix}, \tag{7}$$

where parameters c_k modify system's zeros. Parameters c_k together with a_k modify poles of the system.

The primary goal is to a achieve zero pole cancellation in the transfer function T_{zr}. Coefficients c_k can be tuned in a way that at least one couple of zeros is placed at position of poles to be compensated $\hat{z}_{1,2} = s_{1,2}$. To place a couple of zeros, the following set of constraints are

$$\Re\{\hat{z}_1^n + c_1\hat{z}_1^{n-1} + \ldots + c_{n-1}\hat{z}_1 + c_n\} = 0, \tag{8}$$

$$\Im\{\hat{z}_1^n + c_1\hat{z}_1^{n-1} + \ldots + c_{n-1}\hat{z}_1 + c_n\} = 0. \tag{9}$$

When a couple of complex zeros is placed the controller exhibits filtering properties. This can be seen in the example of a magnitude frequency response of T_{zr} in Fig. 3, where the drop of the amplitude at the given frequency is shown.

Each constraint (8), (9) removes one degree of freedom of the controller, where total number of degrees is determined by the order of the controller N_c. Therefore, to place one couple of complex conjugated zeros $N_c \geq 2$ and the remaining parameters of c_k and a_k can be used for further purposes, e.g. H_∞ optimization, minimization of the spectral abscissa etc.

The set of equations (8)-(9) can be rewritten into form

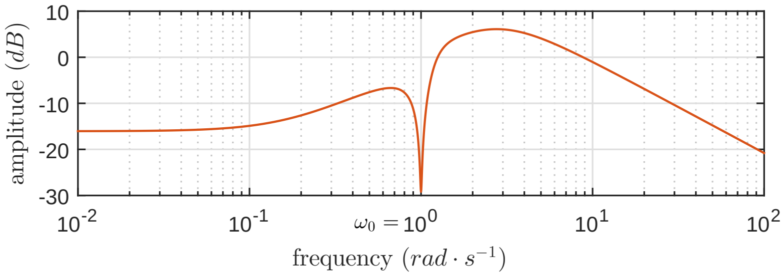

Fig. 3 Magnitude frequency response of a controller designed with partial zero placement

$$HC_c = R, \quad with \quad C_c = [c_1 \quad \cdots \quad c_n]^T, \tag{10}$$

and by applying the singular value decomposition which gives us $H = U\Sigma G^*$, the gains can be then separated in two parts

$$C_c = C_0 + EL \tag{11}$$

where

$$C_c = \underbrace{[g_1|\cdots|g_m]\begin{bmatrix} l_1 \\ \vdots \\ l_m \end{bmatrix}}_{C_0} + \underbrace{[g_{m+1}|\cdots|g_n]}_{E}\underbrace{\begin{bmatrix} l_{m+1} \\ \vdots \\ l_n \end{bmatrix}}_{L} \tag{12}$$

where $l_i = \frac{\bar{e}_i}{\sigma_i}$, $U^*R = [\bar{e}_1 \cdots \bar{e}_m]^T$ and $\Sigma = [diag(\sigma_1 \cdots \sigma_m)]$. A size of the first part of (12) is determined by m which is given by a number of constraints (8)-(9). The remaining degrees of freedom is defined as $n-m$, where n is given by an order of the controller (1). This separation of the controller's coefficients provides elimination of the constraints (8)-(9) which would be very difficult to incorporate into an optimization routine.

Connecting a system (6) and the controller (1), with C_c matrix defined in (12), the following system is obtained

$$\begin{cases} \dot{x}(t) = & A_P x(t) \quad + B_P (C_0 + EL)^T (r(t) - y(t)), \\ \dot{p}(t) = B_c C x(t) & \qquad + A_c (r(t) - y(t)). \end{cases} \tag{13}$$

Now, the remaining parameters in L and a_c are available to modify the closed loop system. To show the functionality of the proposed method, the minimization of the spectral abscissa c is presented here. The spectral abscissa is in general a non-convex function where differentiability may not occur when more than one eigenvalue is active, i.e., an eigenvalue whose real part equals the spectral abscissa [9, 10]. Lipschitz continuity fails when an active eigenvalue is multiple and non-semisimple. On the other hand, the spectral abscissa function is differentiable at points where there is only one active eigenvalue with multiplicity one. Since this is the case with probability one when randomly sampling parameter values, the spectral abscissa is smooth almost everywhere. The above properties exclude classical methods to solve the problem Hybrid algorithm for non-smooth optimization (HANSO) software [11] where combination of BFGS with Wolfe weak line search algorithm is able to solve such problems. The software only requires objective function and its derivatives with respect to controller parameters wherever the objective function is differentiable. The objective is to minimize the spectral abscissa of the closed loop system. Defining a vector of variables $p = [L \quad a_1 \quad \cdots \quad a_n]^T$ of length N_p , the optimization can be then defined as

$$\min_p c(p), \tag{14}$$

where the spectral abscissa is defined as

$$c(p) := \sup\left\{\Re(s) : \ s \in M(s;p)\right\}, \tag{15}$$

with

$$M(s;p) := \left\{s \in \mathbb{C} : \det\left(sI - \mathscr{A} \quad\right) = 0\right\}. \tag{16}$$

where

$$\mathscr{A} = \begin{bmatrix} A_P & B_P(C_0 + EL)^T \\ -B_c C_P & A_c \end{bmatrix} \tag{17}$$

is the matrix of the closed loop system (13). The software also requires derivatives of the objective function with respect to controller parameters. If only one characteristic root with multiplicity one is active then the spectral abscissa is differentiable and expressed as

$$\frac{\partial c}{\partial p} = \begin{bmatrix} \frac{\partial c}{\partial p_1} & \cdots & \frac{\partial c}{\partial p_{N_p}} \end{bmatrix}^T = \Re\left\{ -\frac{1}{v^* \frac{\partial M}{\partial s} w} \begin{bmatrix} v^* \frac{\partial M}{\partial p_1} w & \cdots & v^* \frac{\partial M}{\partial p_{N_p}} w \end{bmatrix}^T \right\}, \tag{18}$$

where v and w are the left and right eigenvectors corresponding to the rightmost eigenvalue.

It is important to note that the technique used to eliminate controller parameters requires that the polynomial, which determines its zeros, linearly depends on the controller parameters. This reduces the applicability to SISO systems. Furthermore a linear dependence also requires that throughput gain D_c is not part of the controller (1).

3 Architectures with input shapers

The technique utilizing the inverse shaper is based on classical reference input shaping. Hereby, this section introduces basics of input shaping with time delays followed by a method with an inverse shaper. The section points out the main advantages of inverse shapers, which will be compared with the technique from Sec. 2.

3.1 Input shaping background

The classical feedforward scheme with input shaper is show in Fig. 1. The main idea is to filter undesirable frequency coming from the reference signal. Filtering with time delays has main advantage that the modified input reference can be preserved as non-decreasing which is hard to achieve with a classical notch filter.

Utilizing time delays every delay-based input shaper can be described with a Stieltjes integral as

$$h(t) = \int_0^T r(t-\mu)d\lambda(\mu) \tag{19}$$

where $r,u \in R$ are the input and the output, respectively. The function $\lambda(\mu)$ is the distribution of the delay over time interval $[0,T], T \in R$ and $T > 0$. The delay can be distributed in a different ways. The distribution can be made of discrete delays and the shaper then has the form

$$h(t) = A_0 r(t) + \sum_{k=1}^{N} A_k r(t-\tau_k) \tag{20}$$

where $A_k \in R$ are gains and $\tau_k > 0$ are delays. The classical Posicast (also called ZV) shaper has one only delay and has form $h(t) = A_0 r(t) + A_1 r(t-\tau)$ with first, non-delayed parameter $0 < A_0 < 1$ and the gain for the delayed part $A_1 = (1-A_0)$. For more general discrete distributions see [12].

The distribution of the delay can also be continuous. Then the shaper is in the form

$$h(t) = A_0 r(t) + \sum_{k=1}^{N} A_k \int_0^{\tau_k} g_k(\mu) r(t-\mu) d\mu, \tag{21}$$

where the $g_k(\mu)$ functions can be linear (equally distributed delay, the shaper is then called DZV [13, 14] or even more complicated distributions as shown in [15]). The delay distributed with smooth polynomial functions is described in [16]. The shaper in form (21) has retarded spectrum whereas the shaper with discrete delays (20) has undesirable neutral spectra ([17]). Neutrality brings difficulties in dynamic analysis and requires special attention in the feedback design [18, 19], which would be crucial in method using an inverse shaper. For this reason, we focus only on shapers with retarded dynamics.

3.2 Inverse shaper

The closed loop architecture with inverse shaper proposed in [3] is show in Fig. (4). The shaper S has spectrum consisting only of zeros and the inversion of the shaper S^{-1} turns its zeros into the poles of transfer function $\frac{1}{S(s)}$. Of course, this mathematical operation is only possible when the transfer function is proper. In case of shapers in form of (20) or (21) the inversion always exists if $A_0 > 0$.

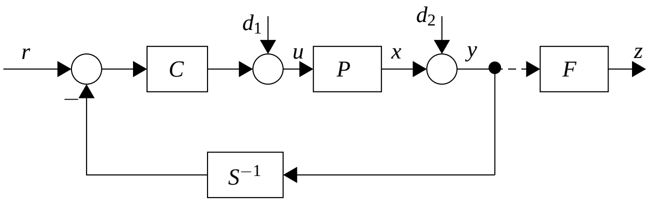

Fig. 4 Block scheme with inverse shaper in the feedback

The main idea of including the inverse shaper in the feedback is to project its filtering properties in the transfer functions from all possible inputs. For reference input as

$$T_{zr} = \frac{CP}{1+CP\frac{1}{S}}F = \frac{\frac{C_N P_N}{C_D P_D}}{\frac{C_D P_D S + C_N P_N}{C_D P_D S}} \frac{F_N}{F_D} = \frac{C_N P_N S}{C_D P_D S + C_N P_N} \frac{F_N}{F_D} \tag{22}$$

and for the input d_1 and output d_2 disturbances

$$T_{zd_1} = \frac{P}{1+CP\frac{1}{S}}F = \frac{\frac{P_N}{P_D}}{\frac{C_D P_D S + C_N P_N}{C_D P_D S}} \frac{F_N}{F_D} = \frac{C_D P_N S}{C_D P_D S + C_N P_N} \frac{F_N}{F_D} \tag{23}$$

$$T_{zd_2} = \frac{1}{1+CP\frac{1}{S}}F = \frac{1}{\frac{C_D P_D S + C_N P_N}{C_D P_D S}} \frac{F_N}{F_D} = \frac{C_D P_D S}{C_D P_D S + C_N P_N} \frac{F_N}{F_D} \tag{24}$$

As can be seen in the transfer functions (22)-(24), infinitely many zeros of the shaper S appear in all numerators of the transfer functions. Then the dominant zeros can be used to compensate the oscillatory modes of $F(s)$. This means, that neither a reference change, nor an input or input disturbance excite the given frequency. On the other hand, the quasi-polynomial form of the shaper also appear in the denominator of the transfer functions and projects its zeros into poles of the system spectra.The main advantage of the shapers with retarded spectra is revealed now. If the shaper has retarded spectra also the closed loop with inverse shaper has retarded spectra and stability issues with small delay perturbations (see, [18, 19]) are omitted.

The closed loop system with the inverse shaper can be unstable or relatively slow. To stabilize the system or modify system spectra, a fixed-order controller can be used to perform the tasks. A fixed-order controller design for infinite dimensional system allows to obtain relatively simple controller and the order of the controller does not necessary need to be the same as the open-loop system. As in the previous, shaper-free, case the design of the controller can be executed for various objective functions, e.g. minimizing the spectral abscissa or H-infinity norms. For the fixed-order controller and the mentioned objectives, the optimization problem is in general non-convex, non-smooth etc. Such problems can once again be handled by recently developed non-smooth, non-convex optimization techniques that are implemented in the package HANSO.

To demonstrate the applicability of the scheme with an inverse shaper, one specific shaper is chosen and applied to the system. The system is then optimized in sense of minimizing the spectral abscissa.

Firsty, we define the input shaper which will be used. The shaper is in form

$$h(t) = ar(t) + (1-a)\int_0^T r(t-\mu)d\lambda(\mu) \tag{25}$$

where r and h are the shaper input and output, respectively, $a \in R^+, a < 1$ is the gain parameter, and the distribution of the delays is prescribed by the non-decreasing function $\lambda(\mu)$. Considering that the overall delay consists of a series of lumped and equally distributed delay of the lengths T the input shaper has transfer function

$$S_{DZV}(s) = a + (1-a)\frac{1-e^{-sT}}{Ts}e^{-s\tau} \tag{26}$$

which consists of lumped delay τ and equally distributed delay of the length T. The interpretation of the given transfer function is shown in the time domain in Fig. 5 and in frequency domain in Fig. 6, where its filtering properties for the given nominal frequency $\omega_0 = 1rad/s$ is obvious.

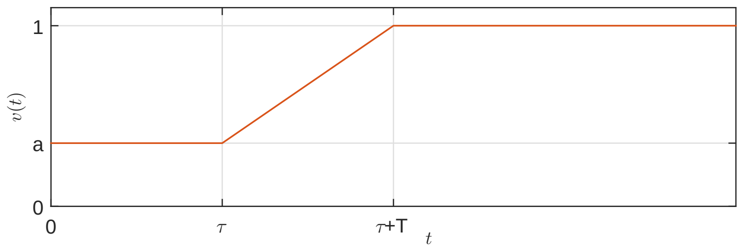

Fig. 5 Step response of the shaper

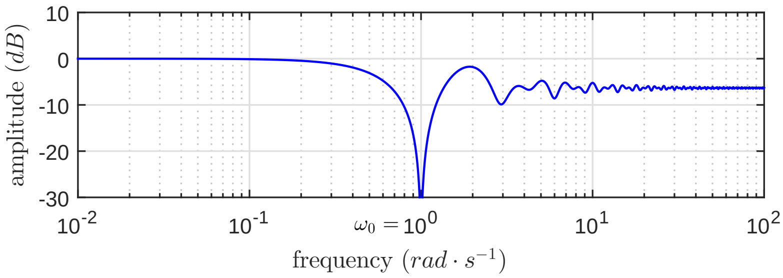

Fig. 6 Magnitude frequency response of the shaper

The inversion of the shaper is realized by the following formula

$$h(t) = \frac{1}{a}(y(t) - (1-a)b(t)) \tag{27}$$

where

$$b(t) = \frac{1}{\tau}\int_{t-(T+\tau)}^{t-\tau} h(\mu)d\mu \tag{28}$$

which can be implemented as a dynamic equation

$$\dot{b}(t) = \frac{1}{T}\left(h(t-T) - h\left(t-(T+\tau)\right)\right). \tag{29}$$

Connecting equations for shaper (27) and (29) together with the system (6) the system can be described by a set of Delayed Differential and Algebraic Equations (DDAEs) as

$$\begin{cases} \dot{x}(t) = A_P x(t) + B_P u(t), \\ h(t) = \frac{1}{a}C_P x(t) - \frac{1-a}{a}b(t), \\ \dot{b}(t) = \frac{1}{T}h(t-T) - \frac{1}{T}h(t-(T+\tau)). \end{cases} \tag{30}$$

The next task is to design a controller that stabilizes and optimizes the system. To get a good comparison with the shaper-free method the spectral abscissa is the objective function for both cases. The controller is in form (1) with no requirements on structure as in shaper-free method. The vector of variables is here defined as $q = vec\begin{bmatrix} A_c & B_c \\ C_c & 0 \end{bmatrix}$ with length N_q. The spectral abscissa for the closed loop is defined as

$$c(q) := \sup\left\{\Re(s) : \ s \in M_S(s,q)\right\}, \tag{31}$$

with

$$M_S(s,q) := \left\{ s \in \mathbb{C} : \det\left(s\mathscr{E} - \mathscr{A}_0 - \mathscr{A}_1\left(e^{-sT} + e^{-s(T+\tau)}\right)\right) = 0 \right\}. \tag{32}$$

where

$$\mathscr{A}_0 = \begin{bmatrix} A_P & -B_P D_c & 0 & B_P Cc \\ \frac{1}{a}C_P & -1 & -\frac{1-a}{a} & 0 \\ 0 & 0 & 0 & 0 \\ B_c C & 0 & 0 & A_c \end{bmatrix}, \mathscr{A}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{1}{T} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{33}$$

The function of spectral abscissa has the same properties as in previous shaper-free case and HANSO software can be chosen to handle the problem. Also here, derivatives are necessary for the optimization and the same rules for derivatives as in (18) applies.

4 Numerical simulations

Consider the mechanical system depicted in Fig. 7 where the primary structure is a 2DOF system described by

$$A_P = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k}{m_1} & \frac{k}{m_1} & -\frac{c}{m_1} & \frac{c}{m_1} \\ \frac{k}{m_2} & -\frac{k}{m_2} & \frac{c}{m_2} & -\frac{c}{m_2} \end{bmatrix}, B_P = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix} \tag{34}$$

with the parameters given in Table 1. The secondary structure is attached to the primary structure by a spring and a damper. We assume that mass $m_a << m_2$ and therefore the dynamic of the primary structure may be considered as decoupled from the dynamics of the secondary structure. The oscillatory mode of the secondary structure is defined by its natural frequency $\omega = \sqrt{\frac{k_a}{m_a}}$ and damping ratio $\zeta = \frac{c_a}{2\sqrt{m_a k_a}}$ as $\bar{s}_{1,2} = -\beta \pm j\Omega$, where $\beta = \omega\zeta, \Omega = \omega\sqrt{1-\zeta^2}$.

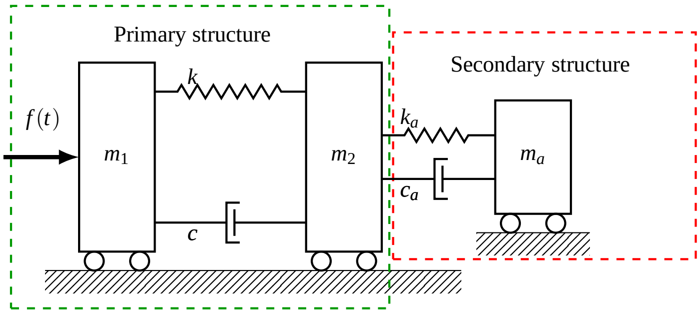

Fig. 7 Mechanical scheme of 2 degrees of freedom primary structure with attached secondary structure. Note that we neglect the coupling between primary and secondary structureas $m_2 << m_a$.

Table 1 Parameters of the system

m_1	c	k	m_2	m_a	c_a	k_a	ω	ζ
1 kg	10 kg s^{-1}	1000 N m^{-1}	10 kg	1 kg	1 kg s^{-1}	1 N m^{-1}	1 rad s^{-1}	0.01

The both cases, shaper-free scheme and scheme with inverse shaper, are designed for the same system. The results are compared in both the compared in frequency and time domain. The results can be seen in the Fig. 8, where zeros and poles of the system are shown. As shown, the initial system without a controller is at the stability boundary with a double pole at the origin and has no zeros. Closing the feedback with designed controller makes the system stable with required zeros in the transfer function from reference to system output (5). In case with the inverse shaper, the zeros of the shaper merge with poles of the system and introduce its dynamics into the closed loop.

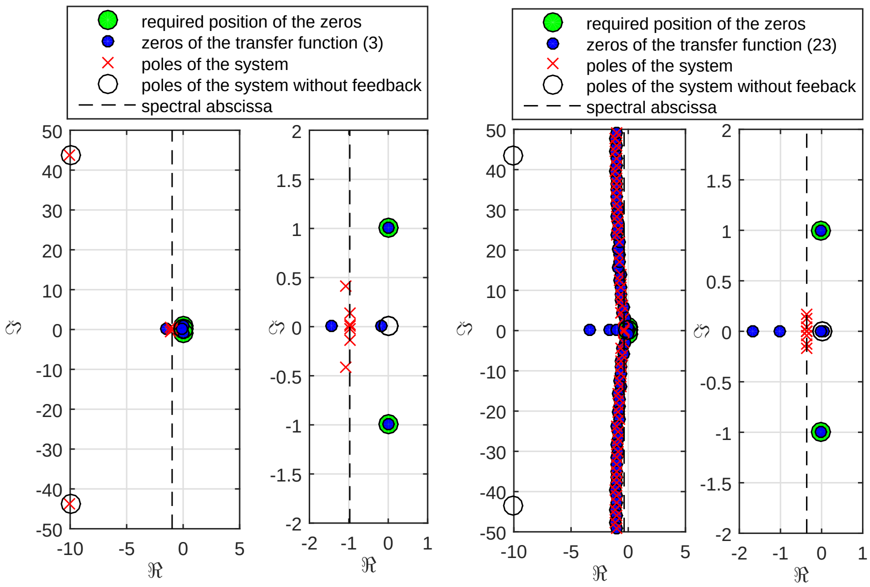

Fig. 8 Spectra of the shaper-free (left) and shaper-based (right) systems.

Unfortunately, in case without inverse shaper the couple of assigned zeros appear only in this particular transfer function (5) but not in ones coming from the disturbances $d_{1,2}$. Zeros for different transfer functions are compared in Fig. 9. For the case with shapers, part of infinitely many zeros are shown. In fact, the spectrum is retarded, hence the chain of zeros have real parts moving off to minus infinity.

As shown in Fig. 10, the oscillations do not appear when the reference signal is changed but appear when one of the disturbances are present. This behaviour disappear when the inverse shaper is applied in the feedback. Note, that, the response to the disturbance has non-zero steady state error. The error could be eliminated by putting another constraints on the controller or implementing additional integrator. Another advantage of the inverse shaper scheme is the tendency to form smoother responses, even though the desirable monotonous responses have not been achieved in this particular example due to 'fast' controller setting.

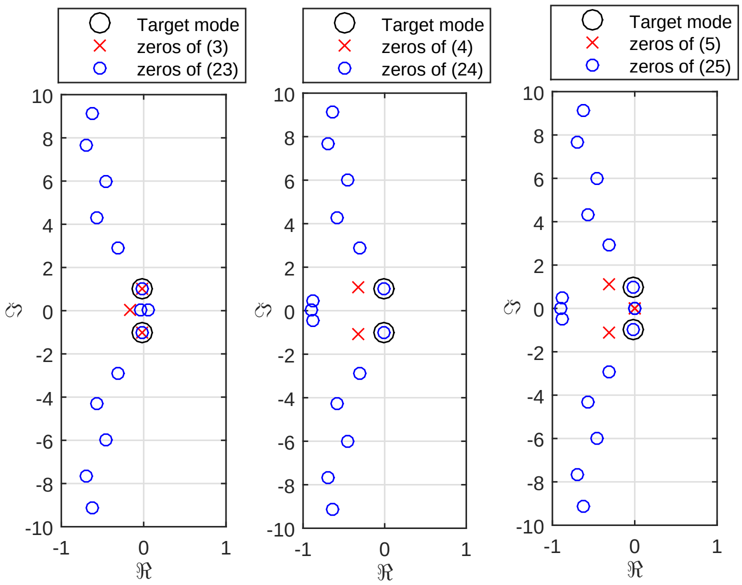

Fig. 9 Comparison of zeros of: left-from reference, middle-from input disturbance d_1, right-from output disturbance d_2

5 Summary

We shown how certain constraints can help to construct controllers with properties mimicking filtering properties of the inverse shaper. This approach allows only certain channels, either a single reference input or a couple of input and output disturbances, to have filtering properties, and applies only to SISO system only.Moreover, compared to the input shaping, the responses do not tend to have desirable monotonous character.

In order to address both the set-point and disturbance cases, the controller would need to be separated into two blocks analogously as it is done in Fig. 4. Instead of the inverse shaper, one can place the filter with the flexible mode as its poles. By this option however, the monotonicity of the response from the set-point changes cannot be achieved neither.

The inverse shaper introduces additional disturbance rejections without exciting oscillatory modes of the flexible structure. On the other hand, infinitely many zeros of the shaper turn into poles of the system and make design of the controller more complicated. Stability issues of neutral systems are removed by using shapers with distributed time delay, whereas the shaper-free method does not need any special stability treatment.

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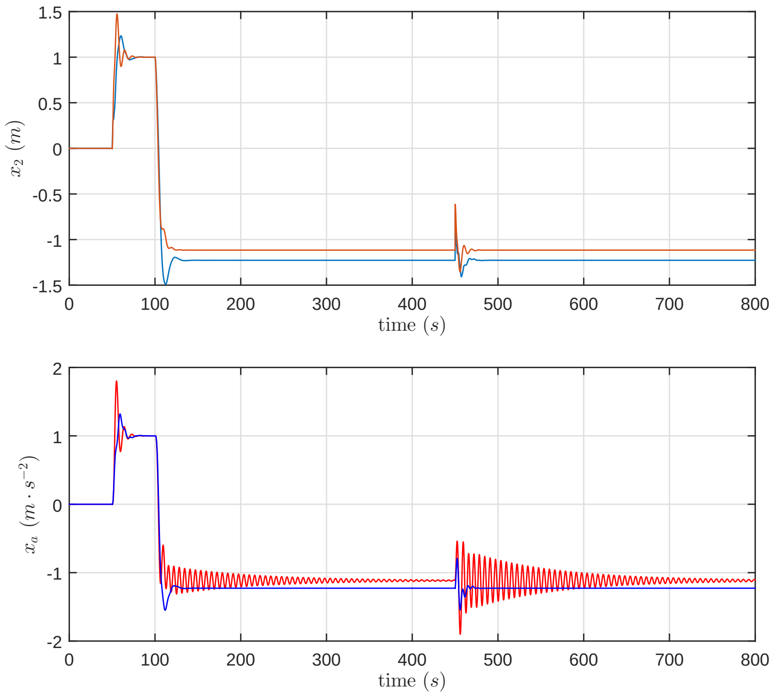

Fig. 10 Red-Response of the system without inverse shaper, designed with partial zero placement; Blue- Response of the system with inverse shaper. Response to the chane of reference r at time $t = 5s$, input disturbance d_1 appears at time $t = 100s$ and output disturbance d_2 at time $t = 450s$. The upper figure shows the position of the second cart x_2 and the lower figure shows the position of the flexible structure x_a

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